

KNO scaling 30 years later

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KNO scaling, i.e. the collapse of multiplicity distributions P_n onto a universal scaling curve manifests when P_n is expressed as the distribution of the standardized multiplicity $(n - c)/\lambda$ with c and λ being location and scale parameters governed by leading particle effects and the growth of average multiplicity. At very high energies, strong violation of KNO scaling behavior is observed ($p\bar{p}$) and expected to occur (e^+e^-). This challenges one to introduce novel, physically well motivated and preferably simple scaling rules obeyed by high-energy data. One possibility what I find useful and which satisfies the above requirements is the repetition of the original scaling prescription (shifting and rescaling) in Mellin space, that is, for the multiplicity moments' rank. This scaling principle will be discussed here, illustrating its capabilities both on model predictions and on real data.

*Dedicated to Wolfram Kittel
on the occasion of his 60th birthday*

data points $P_n(s)$ measured at different energies s collapse onto the unique scaling curve $\psi(z)$. After Koba [3], let me illustrate this schematically:

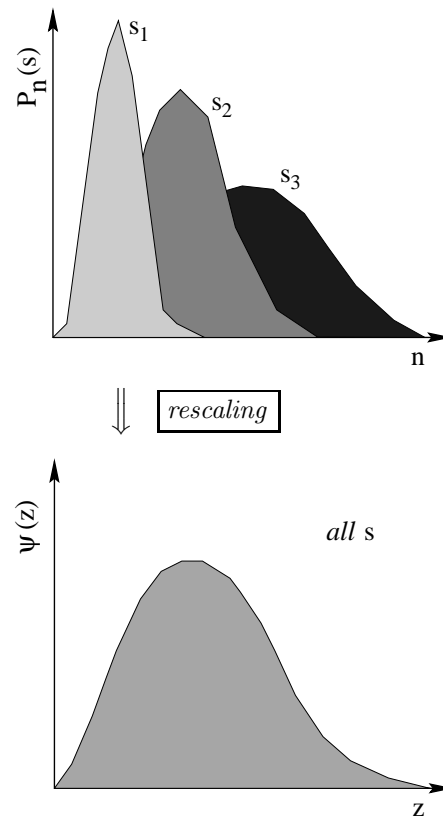
1. THE EARLY YEARS

1.1. Polyakov and Koba-Nielsen-Olesen

In the Millennium Year we celebrated the 30th anniversary of the famous Eq. (1), the basic result of Polyakov and of Koba, Nielsen and Olesen concerning the asymptotic behavior of the multiplicity distributions [1–3]. These authors put forward the hypothesis that at very high energies s the probability distributions $P_n(s)$ of producing n particles in a certain collision process should exhibit the scaling (homogeneity) relation

$$P_n(s) = \frac{1}{\langle n(s) \rangle} \psi\left(\frac{n}{\langle n(s) \rangle}\right) \quad (1)$$

as $s \rightarrow \infty$ with $\langle n(s) \rangle$ being the average multiplicity of secondaries at collision energy s . This so-called KNO scaling hypothesis asserts that if we rescale $P_n(s)$ measured at different energies via stretching (shrinking) the vertical (horizontal) axes by $\langle n(s) \rangle$, these rescaled curves will coincide with each other. That is, the multiplicity distributions become simple rescaled copies of the universal function $\psi(z)$ depending only on the scaled multiplicity $z = n/\langle n(s) \rangle$. In the picturesque terminology of Stanley [4] the rescaled



The data collapsing behavior is a fundamental property of homogeneous functions. According to Eq. (1), at very high energies the multiplicity distributions $P_n(s)$ are homogeneous functions of degree -1 of n and $\langle n(s) \rangle$. Homogeneity rules play a central role in the theory of critical phenomena [4]. Near the critical point of a physical system the thermodynamic functions exhibit homogeneous form which implies the existence of a law of corresponding states: using a suitably chosen scaling transformation it is possible to bring different states of the same system to coincidence and thus to compress many experimental or theoretical results into a compact form. The KNO scaling hypothesis Eq. (1) is just such a law of corresponding states.

In the celebrated paper [2] Koba, Nielsen and Olesen showed that the validity of Feynman scaling is a sufficient condition for Eq. (1) to hold. It is worth mentioning however that the mathematical rigor of their derivation was criticized by several authors. Moreover, the precise finding of Ref. [2] was the energy independence of the moments $C_q = \langle n^q(s) \rangle / \langle n(s) \rangle^q$ in the limit $s \rightarrow \infty$. The scaling hypothesis Eq. (1) was formulated with the additional assumption that the moments $C_q = \int_0^\infty z^q \psi(z) dz$ determine uniquely the scaling function $\psi(z)$. Polyakov arrived at the asymptotic multiplicity scaling law Eq. (1) two years earlier than Koba, Nielsen and Olesen by formulating a similarity hypothesis for strong interactions in $e^+e^- \rightarrow \text{hadrons}$ annihilation; see later. Somehow this fundamental and elegant work was almost completely ignored in the early 70s and Eq. (1) became known as the KNO scaling hypothesis in the high-energy physics community.

1.2. Modifications

The scaling relation Eq. (1) can hold only approximately since multiplicity n is a discrete random variable whose stretching or shrinking by a scale factor leaves the probabilities P_n unaltered. The proper meaning of Eq. (1) is that with increasing collision energy s the discrete multiplicity distributions P_n can be approximated with increasing accuracy by a continuous probability density function $f(x)$ via $P_n \approx f(x = n)$ (KNO prescription) or by $P_n \approx \int_{x=n}^{x=n+1} f(x) dx$ (called

KNO-G “scaling”) where, in the most popular modification of the original scaling rule, $f(x)$ has the generic Czyżewski-Rybicki form [5,6]

$$f(x) = \frac{1}{\lambda} \psi\left(\frac{x-c}{\lambda}\right) \quad (2)$$

with scale parameter $\lambda > 0$ and location parameter c . Data collapsing behavior is observed if the only s -dependent parameters of the approximate shape function $f(x)$ are c and λ , i.e. if $f(x)$ depends on collision energy only through a change of location and scale in x . The variation of λ with increasing s reflects the growth of average multiplicity, whereas the s -dependence of c is usually associated with leading particle effects (so-called AKNO or KNO- α scaling). Other physical explanations are also possible, see e.g. the contribution of Marek Płoszajczak on Δ -scaling.

2. NEW SCALING PRINCIPLE

Taking into account a possible location change (shift) in multiplicity rarely proved to be sufficient to restore the data collapsing behavior of P_n when the original scaling law Eq. (1) is violated at very high energies (hence we use $c = 0$). Nevertheless, the principle of scale *and* location change in the shape of multiplicity distributions can be extremely powerful through the following generalization of the KNO scaling rule. Consider that the multiplicity distributions, more precisely the continuous probability densities $f(x)$ approximating P_n generate s -dependent shifting and rescaling in Mellin space. The Mellin transform of a probability density $f(x)$ is defined by $\mathcal{M}\{f(x); q\} = \int_0^\infty x^{q-1} f(x) dx$ and it provides the $(q-1)$ th moment $\langle x^{q-1} \rangle$. Via the functional relation

$$\mathcal{M}\left\{x^r f(x^\mu); q\right\} = \frac{1}{\mu} \mathcal{M}\left\{f(x); \frac{q+r}{\mu}\right\} \quad (3)$$

one can introduce translation and dilatation in the moments’ rank q by performing the transformation $f(x) \rightarrow x^r f(x^\mu)$ of the probability densities approximating the shape of P_n . In the followings I shall illustrate the utility of performing a location or scale change in Mellin space; for further details, see Refs. [7–9].

3. TRANSLATION IN \mathcal{M} -SPACE

3.1. Motivation

There are several possible dynamical explanations of a shift in Mellin space. The basic idea is simply the following. Recall that in Eq. (1) the normalization of the KNO function reads $\int_0^\infty \psi(z) dz = \int_0^\infty z \psi(z) dz = 1$, that is, $\langle z \rangle = 1$. Assume that violation of KNO scaling is observed and we measure s -dependent “scaling” functions $\psi(z)$. In order to arrive at data collapsing behavior of P_n the simplest possibility is to rescale the function $\varphi(z) = z \psi(z)$ by its first moment given by C_2 . In terms of P_n and the moments $\langle n^q \rangle$ this corresponds to rescaling the distribution $P_{n,1} = n P_n / \langle n \rangle$ by its mean $\langle n \rangle_1 = \langle n^2 \rangle / \langle n \rangle$. The distribution $P_{n,1}$ is known in the mathematical literature as the first *moment distribution* of P_n . If the rescaling of $P_{n,1}$ by $\langle n \rangle_1$ is not sufficient to arrive at data collapsing behavior, one can try the same recipe for the higher-order moment distributions $P_{n,r} = n^r P_n / \langle n^r \rangle$. The generalization is straightforward. Let me show you now an example when Eq. (1) is clearly violated, but performing a shift in Mellin space can restore the collapse of different P_n curves onto a single scaling curve.

3.2. Intermittency and multifractality

The most obvious example is intermittency. As it turned out during the past 15 years, multiparticle production often yields scale-invariant density fluctuations [10–12]. This can be observed through the power-law scaling $C_q \propto \delta^{-\varphi_q}$ of the normalized moments $C_q = \langle n^q \rangle / \langle n \rangle^q$ of $P_n(\delta)$ as the bin-size δ in phase-space is varied (for clarity, we neglect here the influence of low count rates). Assuming that the fluctuations show monofractal structure, the so-called intermittency exponents φ_q are given by $\varphi_q = \varphi_2(q-1)$ and the anomalous fractal dimensions $D_q = \varphi_q / (q-1)$ are q -independent, $D_q = D_2$. The normalized moments C_q of $P_n(\delta)$ take the form

$$C_q = A_q [C_2]^{q-1} \quad \text{for } q > 2 \quad (4)$$

with coefficients A_q independent of bin-size δ [12]. Of course Eq. (1), with $s \rightarrow \delta$, is violated since we observe $C_2 \propto \delta^{-\varphi_2}$.

In the restoration of data collapsing behavior of P_n for self-similar fluctuations the basic trick is the investigation of the higher-order moment distributions defined before. Their moments are $\langle n^q \rangle_r = \langle n^{q+r} \rangle / \langle n^r \rangle$, that is, the moments of the original P_n are transformed out up to r th order via performing a shift in Mellin space, see Eq. (3). For $r = 1$ the normalized moments of the first moment distribution $P_{n,1}$ are found to be $C_{q,1} = C_{q+1} / [C_2]^q$ in terms of the original C_q and comparison to Eq. (4) yields $C_{q,1} = A_{q+1}$ for monofractal multiplicity fluctuations. Since the coefficients A_q are independent of bin-size δ , we see that monofractality yields not only *power-law scaling* of the normalized moments of P_n but also *data collapsing* behavior of the first moment distributions $P_{n,1}$ measured at different resolution scales δ . Increasing the (possibly fractional) rank r of the moment distributions allows the restoration of data collapsing behavior in the presence of increasing degree of multifractality of scale-invariant multiplicity fluctuations [7,8]. The effect of low multiplicities (Poisson noise) can be taken into account via the study of *factorial* moment distributions $P_{n,r} = n^{[r]} P_n / \langle n^{[r]} \rangle$ and their factorial moments.

3.3. Ginzburg-Landau phase transition

In recent years considerable interest has been devoted to the Ginzburg-Landau theory of the phase transition from quark-gluon plasma to hadronic matter [13,14]. Instead of a strict power-law scaling $F_q \propto \delta^{-\varphi_q}$ of the normalized factorial moments, a so-called *F-scaling* behavior $F_q \propto [F_2]^{\beta_q}$ is obtained with $\beta_q = (q-1)^\nu$. For second-order transition $\nu = 1.304$ and in case of a first-order transition $\nu = 1.45$. The *F-scaling* rule of the above form makes difficult to identify the family of probability laws P_n belonging to the model. But considering $P_{n,1}$ instead of P_n the corresponding factorial moments turn out to be $F_{q,1} \propto [F_2]^{\beta_q}$ with $\beta_q = q^\nu - q$ and hence [7]

$$F_{q,1} \propto [F_{2,1}]^{\beta_q} \quad \text{with } \beta_q = \frac{q^\nu - q}{2^\nu - 2}.$$

This form of *F-scaling* is the familiar log-Lévy law [15] with stability index $0 < \nu \leq 2$. Thus, via moment shifting, we succeeded in identifying $P_{n,1}$

for the Ginzburg-Landau formalism with a strict bound on the essential parameter ν of the model. The theoretically relevant values are within the allowed range. Although this particular example is not related directly to data collapsing behavior of multiplicity distributions, it illustrates the utility of the moment distributions $P_{n,r} \propto n^r P_n$ which correspond to translation in Mellin space.

4. RESCALING IN \mathcal{M} -SPACE

4.1. Motivation

The most important reason of a possible change of scale in Mellin space comes from QCD. In higher-order pQCD calculations, allowing a more precise account of energy conservation in the course of multiple parton splittings, the natural variable of the multiplicity moments is the *rescaled* rank $q\gamma$ instead of rank q itself [20–22] with $\gamma(\alpha_s)$ being the multiplicity anomalous dimension of QCD. Because of the running of the strong coupling constant α_s , it is inevitable to adjust an energy dependent scale factor in Mellin space if one wants to arrive at data collapsing behavior of the multiplicity distributions $P_n(s)$.

4.2. Running coupling

In the very first paper predicting the multiplicity scaling law Eq. (1), Polyakov constructed a model for the short-distance behavior of strong interactions based on the hypothesis of asymptotic scale- and conformal invariance [1]. When applied to the process of e^+e^- annihilation to hadrons at asymptotically high collision energies, the model predicts that multihadron production goes in a cascade way. First, a few heavy virtual objects, called jets, are formed. These are repeatedly diminished in a branching process until the masses of the subsequent generation jets become at some very late stage comparable to the hadronic masses. This cascading mechanism gives rise to multiplicity distributions $P_n(s)$ satisfying Eq. (1) and the scaling function $\psi(z)$ behaves as $\psi(z) \propto a(z) \exp(-z^\mu)$ with $\mu > 1$ and $a(z)$ being a monomial. Up to a scale factor, $\psi(z)$ is a negative binomial type scaling function, that is, a gamma distribution [16] in the rescaled and power-transformed multiplicity z^μ .

It is worth emphasizing that the above result was achieved years earlier than the emergence of quantum chromodynamics. Interestingly, the predictions of QCD obey some important similarities to the findings of Polyakov:

1) Hadronic jets produced in hard processes are expected in QCD to be self-similar objects composed of subsequent parton branching decays [17]. A highly virtual quark or gluon decays into secondaries which are less off-shell and less energetic. Each of these intermediate decay products splits into novel ones. The subsequent decay chain steps form a scale-invariant branching process which continues until the virtual mass of the quanta becomes approximately 1 GeV.

2) The fragmentation of partons in QCD leads to the typical characteristics of branching processes, in particular, to long-range correlations and to the validity of KNO scaling [18]. Further, it was shown [19] that KNO scaling in e^+e^- annihilation is the consequence of the scale-invariance (asymptotic freedom) in the presence of gluon self-coupling. It is thus the manifestation of the essence of the non-abelian gauge theory of strong interactions.

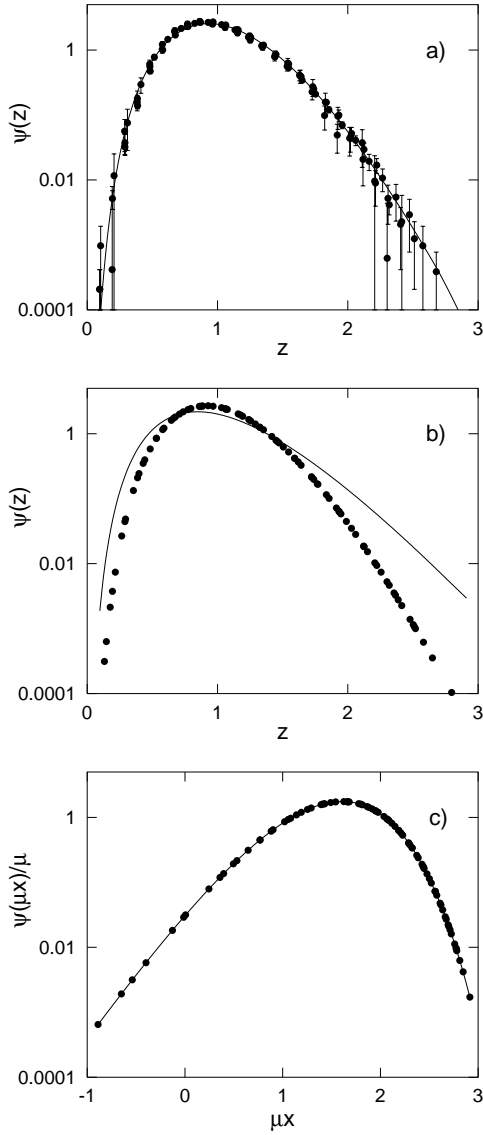
3) Taking into account the higher pQCD effects responsible for energy-momentum conservation in parton jets, the KNO scaling function in e^+e^- annihilation was shown [20] to be identical to Polyakov's form:

$$\psi(z) = \mathcal{N} z^{\mu k - 1} \exp(-[Dz]^\mu) \quad (5)$$

where $k = 3/2$, $\mathcal{N} = \mu D^{\mu k} / \Gamma(k)$, $\mu = (1 - \gamma)^{-1} \approx 5/3$ and D is a scale parameter depending on γ , with γ being the QCD anomalous dimension, $\gamma \approx 0.4$ at LEP1. Hence, the conservation laws strongly reduce the width of the KNO scaling function as compared to the exponential fall-off obtained by lower-order pQCD calculations. The experimental data at $\sqrt{s} = 91.2$ GeV (dots) confirm the prediction (curve) as illustrated in the figure a) below (with the exception that k turns out to be larger, $k \approx 5$).

The crucial difference between the QCD prediction and Polyakov's result lies in the fact that the scaling function $\psi(z)$ given by the Polyakov-Dokshitzer form Eq. (5) *depends* on collision energy s in QCD. That is, QCD predicts violation of

KNO scaling in e^+e^- annihilation which becomes clearly visible at higher energies. This effect is due to the running of the strong coupling constant α_s , hence $\gamma \rightarrow 0$ as $s \rightarrow \infty$, i.e., $\mu \rightarrow 1$ asymptotically [20]. Therefore the tail of $\psi(z)$ widens with increasing s which gives rise to the KNO scaling violation pattern shown in the figure b) below. Asymptotically the exponential fall-off predicted by the double logarithmic approximation is recovered (solid curve) i.e. the negative binomial type scaling form of $P_n(s)$.



Thus we have learned that the pre-QCD and QCD-based descriptions of e^+e^- annihilation to hadrons make essential use of self-similar branching processes, further, both predict the rescaled and power-transformed multiplicity variable z^μ to be gamma-distributed. But in the pre-QCD approach the exponent μ is independent of collision energy s and KNO scaling holds valid, as expected for scale-invariant branching with *fixed coupling*. On the contrary, the QCD-based calculations predict violation of KNO scaling since μ varies with collision energy s , due to the *running* of α_s . Note however that in the latter case data collapsing can be restored in a simple way using *logarithmic* scaling variable. For the Polyakov-Dokshitzer form of $\psi(z)$ given by (5) we have

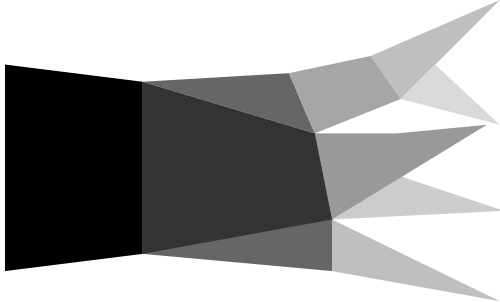
$$\psi(x) = \mu \exp(k\mu x - e^{\mu x})/\Gamma(k), \quad x = \ln(Dz).$$

Since only the exponent μ and scale parameter D of Eq. (5) are expected to depend on collision energy s through the variation of $\gamma(\alpha_s)$, KNO scaling violated by QCD effects is recovered by plotting $\mu^{-1}\psi(\mu x)$ as displayed in the figure c). The scale change in logarithmic multiplicity is governed by the QCD multiplicity anomalous dimension. This particular type of data collapsing of the multiplicity distributions $P_n(s)$ is called log-KNO scaling [8,9] since we have the behavior of type Eq. (2) except that distributions of the logarithmic multiplicity satisfy it.

4.3. Multiplicative cascades

It may turn out that the log-KNO scaling law has ubiquitous appearance in multiparticle dynamics. Random multiplicative cascades play a distinguished role in the dynamics underlying multihadron production, both in soft and hard processes. The multiplicative cascade models like for example the α -model and p -model are in the focus of interest since the pioneering work of Białas and Peschanski [10], see also [12] for a review. Recently, Frisch and Sornette developed a theory of extreme deviations [23] which predicts for multiplication of random variables the generic presence of stretched exponential distributions $f(x) \propto \exp(-\lambda x^\mu)$ with $\mu < 1$. To be specific, consider the product $X_N = m_1 m_2 \cdots m_N$ of independent random variables m_i distributed iden-

tically according to the probability density $p(m_i)$. The above product can arise in a cascade process if primary entities (e.g. particle density) of initial size s_0 are repeatedly diminished in random proportions, so that after N cascade steps we have the product $s_N = s_0 m_1 m_2 \cdots m_N$.



In ref. [23] it is shown that under mild regularity conditions the distribution of the product variable X_N exhibits the asymptotic behavior

$$\mathcal{P}_N(X) \sim [p(X^{1/N})]^N \quad \text{for } X \rightarrow \infty \quad (6)$$

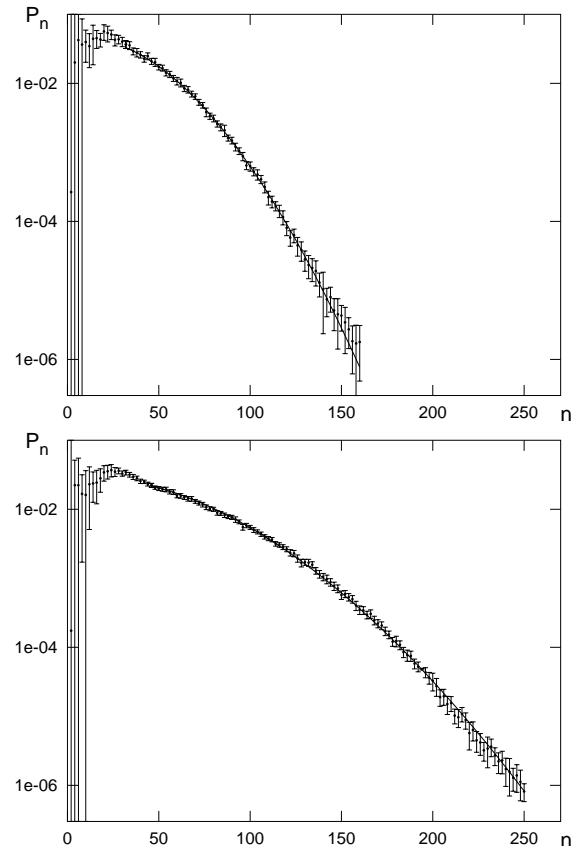
and for finite N . Frisch and Sornette call attention to the intuitive interpretation of (6): the tail of $\mathcal{P}_N(X)$ is controlled by the realizations where all terms in the product are of the same order hence $\mathcal{P}_N(X)$ is, to leading order, just the product of the N densities p , each having the common argument $X^{1/N}$. When $p(x)$ is chosen to be $\propto \exp(-\lambda x^\alpha)$ with $\alpha > 0$, then Eq. (6) leads to stretched exponential tails $\propto \exp(-\lambda N x^{\alpha/N})$ for large N , with stretching exponent $\alpha/N < 1$. Expecting the cascade depth N to be an increasing function of collision energy s , Eq. (1) breaks down but data collapsing can be recovered in the log-KNO manner discussed previously.

5. SCALING AT TEV ENERGIES

The most exciting and challenging task in developing novel scaling relations is testing them on real data. In case of multiplicity distributions the KNO scaling laws Eqs. (1-2) are known to be strongly violated above ISR energies. This violation is most visible for the multiplicity data measured by the E735 Collaboration [24] see also [25] for more details. In the followings I provide a very brief summary of some preliminary results of a study of the E735 data.

5.1. Analysis of E735 multiplicities

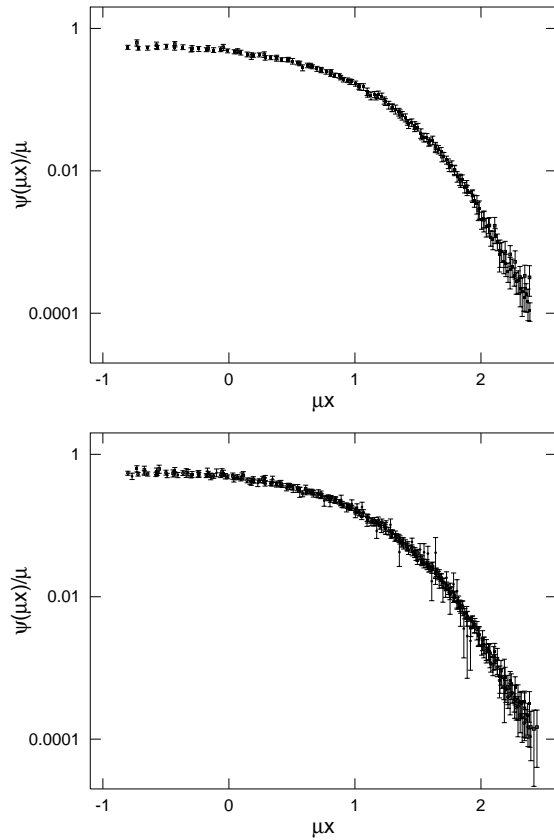
The E735 Collaboration recently published the full phase-space multiplicity distributions in pp and $p\bar{p}$ collisions at c.m. energies $\sqrt{s} = 300, 546, 1000$ and 1800 GeV at Tevatron [24]. The P_n curves corresponding to $\sqrt{s} = 300$ and 1800 GeV are displayed in the following two figures.



It is apparent that bimodal shape of the P_n curves arises at TeV energies, obeying a (not too pronounced) shoulder structure like at SPS. It is argued [24,25] that the low-multiplicity regimes are affected mainly by single parton collisions and exhibit KNO scaling, whereas the large- n tail of the distributions is influenced more heavily by double parton interactions and violate KNO scaling substantially. Another possible explanation of the observed P_n shape is the weighted superposition of a soft and semihard component, the latter one being dominated by minijet production.

These components are responsible of the KNO scaling and KNO-violating regimes of P_n [26,27].

The solid curves in the previous two plots represent the Polyakov-Dokshitzer parametrization Eq. (5) fitted to the KNO-violating $z \geq 1$ component of the E735 data. The quality of fits is reasonably good (for all data sets) as can be depicted from the figures. The fits correspond to $k = \frac{1}{2}$ kept fixed and μ decreasing from $\mu \sim 2.2$ to $\mu \sim 1.7$ as one goes from $\sqrt{s} = 300$ GeV to 1800 GeV. Our findings indicate that the KNO violating component of the E735 multiplicity distributions can be collapsed onto a single scaling curve in the log-KNO manner discussed previously. It is illustrated below.



In the upper figure the $\sqrt{s} = 300$ and 1800 GeV data sets are displayed only, whereas in the lower plot all the four data sets measured by the E735 experiment. The collapse of the data points onto a unique scaling curve is apparent. This very pre-

liminary investigation suggests that the pattern of KNO scaling violation by double parton collisions, or by semihard processes dominated by minijet production, is similar to that of e^+e^- annihilations where log-KNO behavior is expected to arise as well at high energies (see before). But note that in parameter k there is an order of magnitude difference between e^+e^- and $p\bar{p}$ collisions, that is, the scaling functions have different shapes in the two reactions.

One of the earliest reviews [28] of Polyakov's pioneering work concluded with the following speculative note: "Could it be that as in the theory of critical phenomena, while the basic physics (Hamiltonians) between e^+e^- annihilation and hadronic multiparticle production could be entirely different, multiplicity scaling may yet still be a universal feature shared by both?" In the light of our findings the answer to this question is *yes* but the scaling behavior foreseen 30 years ago arises in the logarithm of multiplicity instead of multiplicity itself.

6. SUMMARY

In the past 30 years, data collapsing of multiplicity distributions P_n according to the generic KNO scaling relation Eq. (2) was one of the most extensively studied topics in soft physics. Unfortunately, more and more experimental and theoretical results confirm that Eq. (2) is too simple to be true. The information content of P_n can be represented in an equally convenient manner making use of the multiplicity moments. For example, validity of Eq. (2) corresponds to constancy of the normalized central moments for appropriately chosen c and λ . What can we do if the KNO scaling rule (2) breaks down? How data collapsing behavior of P_n can be restored, if it is possible at all? The approach presented here is extremely simple: let's perform the original scaling transformation (shifting and rescaling) in the multiplicity moments' rank, that is, in Mellin space, rather than in multiplicity.

The functional relation Eq. (3) tells how things transform at the level of P_n . Translation in Mellin space (moment shifting) corresponds to changing from P_n to the moment distributions

$P_{n,r} \propto n^r P_n$. This makes possible to arrive at data collapsing for intermittency phenomena: e.g. $P_{n,1}$ scales for monofractal fluctuations, $P_{n,2}$ for bifractals and so on. Dilatation in Mellin space (rescaling of moments' rank) corresponds to the transformation $P_n \rightarrow P_m$ with $m = n^\mu$. Therefore data collapsing arises in rescaled logarithmic multiplicity (log-KNO). Such behavior is expected to occur for phenomena governed by running coupling effects or in multiplicative cascades of varying length. Considering the physical significance of the mentioned examples in multiparticle dynamics, it will be interesting to see how the proposed scaling behaviors show up in the experiments of the Third Millennium.

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